2023

PHYSICS — HONOURS

Paper : DSE-A-2.1

[Nanomaterials and Applications]

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***any five*** questions : 2×5

(a) Calculate the energy in eV corresponding to room temperature (300 K).

(b) What are the differences between an amorphous material and a single crystal?

(c) Show mathematically that the surface to volume ratio of a nanoparticle is much higher than that of the identical material in bulk.

(d) Why does an electron microscope have more resolving power than an optical microscope?

(e) Why does a colloid of nano-gold appear wine-red in colour?

(f) An electron is confined in a one-dimensional box of length 5 Å. If the electron makes a transition from the first excited state to the ground state, calculate the frequency of the emitted photon. [Mass of the electron = 9.31×10^{-31} kg.]

(g) The cubic lattice of MnO has lattice constant 4.426 Å. Calculate the inter-planar spacings for the (111) and (211) planes.

Group - B

2. Answer ***any three*** questions : 3×5

(a) (i) What do you mean by density of states?

(ii) Show that the density of states for a free particle of mass m in one dimension varies inversely as the square root of its energy. Plot the variation graphically. 1+(3+1)

(b) (i) What are the intermediate steps associated with the growth of thin films by molecular beam epitaxy (MBE) method?

(ii) Write down the basic differences between the techniques of physical vapour deposition (PVD) and chemical vapour deposition (CVD). 3+2

(c) (i) Distinguish between 'direct band gap semiconductor' and 'indirect band gap semiconductor' using E-K diagram.

(ii) X-ray data is taken using a chromium anode $\left(\lambda_{Cr} = 2.289\,\text{Å}\right)$. If the spectrum has a line at $2\theta = 45.4^{\circ}$, what would be the equivalent line position (2θ) for a copper anode $\left(\lambda_{Cu} = 1.5421\,\text{Å}\right)$? 2+3

(d) (i) What are the differences between Frenkel defects and Schottky defects?

(ii) Show that the density 'n' of Schottky defects in a crystal having 'N' atoms is given by

$$n \approx N \exp\left(-\frac{E_v}{2k_B T}\right),$$

where E_v is the energy required to take an ion from a lattice site inside the crystal to a lattice site on the surface and T is the temperature. 2+3

(e) (i) What are the basic differences between optical and electrical band gap?

(ii) Explain the concept of blue shift observed in nanomaterials. 2+3

Group - C

Answer ***any four*** questions.

3. (a) For a rectangular potential barrier

$$V(x) = V_0 \quad \text{for } 0 \le x \le a$$
$$= 0 \quad \text{otherwise}$$

Show that approximate expression for transmission coefficient T is

$$T = \frac{16\left(V_0 - E\right)}{V_0^2} e^{-2Ka},$$

where E is the energy of the particle and $E < V_0$ and $K^2 = \frac{2m\left(V_0 - E\right)}{\hbar^2}$.

(b) What do you mean by quantum confinement? Calculate the exciton Bohr radius of GaAs using the following data :

Dielectric constant of GaAs = 12.4

Effective mass of electron = 0.067 m_0

Effective mass of hole = 0.45 m_0 where $m_0 = 9.1 \times 10^{-31}$ kg. 6+(2+2)

4. (a) What are the different factors affecting the synthesis of a nanomaterial?

(b) Discuss the basic features of top-down and bottom-up processes in the context of synthesis of nanomaterials. Give suitable examples.

(c) What are the advantages of electron beam evaporation? 2+(4+2)+2

5. (a) What is the full form of AFM? What are the various types of forces acting between tip and surface of the sample in AFM?

(b) What are the basic components of AFM? Discuss their functions.

(c) What are the advantages of an AFM? (1+3)+4+2

6. (a) How can the magnetic properties of a material be tailored with the reduction in size?

(b) What do you mean by magnetic quantum well?

(c) What are the basic differences between NEMS and MEMS?

(d) "Incorporation of nanostructured materials can potentially improve the efficiency of a standard solar cell." — Interpret the statement. 2+2+2+4

7. (a) What is thermoionic emission?

(b) Show that the current density 'J' of thermally escaped electron in the direction perpendicular to the heated metal surface is given by

$$J = AT^2 e^{-\frac{W}{K_B T}}$$

where W is the work function of the metal surface, A is a constant, and K_B is the Boltzmann constant.

(c) Show the variation of 'J' with 'T' graphically.

(d) What is the dimension of the constant 'A'? 2+5+2+1

8. Write short notes on : 5×2

(a) Single electron transistor

(b) Single-wall carbon nanotube.

Paper : DSE-A-2.2

(Advanced Classical Dynamics)

Full Marks : 65

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Group - A

1. Answer ***any five*** questions : 2×5

(a) Write the Lagrangian of a particle of mass 'm' moving in a central potential and show that its angular momentum is conserved.

(b) Write down Hamilton's equations of motion using Poisson bracket.

(c) Set up the Hamiltonian of a system for which the Lagrangian is given by

$$L(x, \dot{x}) = \frac{1}{2}\dot{x}^2 - \frac{1}{2}\omega^2 x^2 - \alpha x^3 + \beta x\dot{x}^2,$$

where α and β are constants.

(d) Consider a particle moving in one-dimension under the action of a potential

$$V(x) = x^5/5 - 3x^4/4 + 2x^3/3.$$

Find the stable equilibrium point of the particle, if any.

(e) Consider a thin uniform rectangular sheet of mass M extending from $x = 0$ to $x = a$ and $y = 0$ to $y = b$. Calculate I_{xy}.

(f) Show that the following transformation is canonical :

$$P = \frac{1}{2}\left(p^2 + q^2\right), \quad Q = \tan^{-1}\left(\frac{q}{p}\right).$$

(g) Define a dissipative dynamical system. Explain with an example.

Group - B

2. Answer ***any three*** questions :

(a) For a thin uniform square plate of side 'a' and mass 'm', derive principal moments of inertia. Also calculate the moment of inertia about a diagonal. 3+2

(b) Write down the Lagrangian for a spherical pendulum (a point mass attached to a fixed point by a light inextensible string and moves under the action of gravity). Find the momenta. Find the Hamiltonian. 1+2+2

(c) Two point masses m_1 and m_2 are at rest on a smooth horizontal surface. The masses are connected by a light elastic spring (of spring constant k) of length ℓ_0 (when the masses are at rest). Assuming the motion in one-dimension, write the T and V matrices for small amplitude oscillations of the coupled system. Find the normal frequencies and interpret their significance. 2+2+1

(d) Distinguish between stable and unstable fixed points with the help of flow diagrams. Characterize the fixed points of $\dot{x} = x - x^2$ using linear stability analysis. 2+3

(e) Find the fixed points for the map $x_{n+1} = x_n^2$ and determine their stability. 2+3

Group - C

Answer ***any four*** questions.

3. (a) Show that for a particle with zero initial velocity, the path of minimum time is a cycloid in a brachistochrone.

(b) Consider a function $z = f(x, y)$. Use the Legendre transformation to form $w = g(u, v)$,

where $u = \frac{\partial f}{\partial x}$ and $v = \frac{\partial f}{\partial y}$. 6+4

4. (a) Consider a system having n degrees of freedom. Assuming the potential energy has a minimum, write down its general form in harmonic approximation. How would it look like for a single d.o.f system?

(b) Two identical particles of mass m lying on a frictionless horizontal channel as shown in the figure  where the springs are also identical with spring constant k. Show that the characteristic frequencies of normal modes of oscillation are given by $\left[\left(\frac{3 \pm \sqrt{5}}{2}\right)\frac{k}{m}\right]^{1/2}$.

(c) Two identical particles are attached firmly at the points of trisection of a stretched horizontal string. The particles execute small amplitude transverse oscillations. Schematically indicate the positions of the particles when they execute (i) in-phase, and (ii) out-of phase oscillations (no mathematics required). (2+1)+5+2

5. (a) Justify that a rigid body can have at most six degrees of freedom.

(b) The density of a solid sphere of radius 'a' varies with the radial distance as $\rho(r) = \rho_0(1 + r/a)$, where ρ_0 is a constant and 'r' is the distance from the centre of the sphere. Find the Cartesian components of moment of inertia tensor in a coordinate system whose origin lies at the centre of the sphere. Hence write the ellipsoid of inertia.

(c) Given that (in some units) $I_{xx} = 2.0 = I_{yy}$, $I_{zz} = 5.0$, $I_{xy} = 3.0 = I_{yx}$, other components are zero. Suppose that the rigid body is rotating with angular velocity $\vec{\omega}$ about an axis along $\hat{n} = \left(\hat{i} + \hat{j}\right)/\sqrt{2}$. Find the expression for the moment of inertia about the axis of rotation and hence the kinetic energy. 2+(3+1)+(2+2)

6. (a) Consider a forced Van der Pol oscillator $\ddot{x} - \mu\left(x^2 - 1\right)\dot{x} + \beta x = f \cos\omega t$ with $\mu > 0$.

(i) Cast this as a non-autonomous dynamical system and mention its dimensionality.

(ii) Also write it as an autonomous dynamical system. What is its dimension?

(iii) Derive the conditions for which the system would be conservative, dissipative and anti-dissipative, in the absence of the external forcing.

(b) Consider the Lotka-Volterra model : $\dot{x} = \alpha x - \beta xy$ and $\dot{y} = -\gamma y + \delta xy$, where α, β, γ and δ are all positive constants. Calculate the divergence of the flow vector. (2+2+3)+3

7. (a) Consider the logistic map $x_{n+1} = rx_n(1 - x_n)$ for $0 \le x_n \le 1$ and $0 \le r \le 4$. Find all the fixed points.

Use the transformation $x_n = \sin^2\theta_n$ in the above map for $\left(0 \le \theta \le \frac{\pi}{2}\right)$ to show that the transformed logistic map can be cast in the form of a tent map given by

$$y_{n+1} = \begin{cases} 2y_n & \text{if } 0 \le y_n \le \frac{1}{2} \\ 2\left(1 - y_n\right) & \text{if } \frac{1}{2} \le y_n \le 1 \end{cases}$$

for $y_n = \frac{2\theta_n}{\pi}$. Draw this map and its first iterate.

(b) Draw the phase portrait of the dynamical system given by $\dot{x} = 1 - 2\cos x$ in the range $0 \le x \le 2\pi$. Hence find the fixed points and comment on their stability. (2+2+2)+(2+2)

8. (a) The Lagrangian of a particle of charge 'q' and mass 'm' moving with a velocity $\vec{v}(t)$ in a region containing both electric field $\vec{E}(\vec{r}, t)$ and magnetic field $\vec{B}(\vec{r}, t)$ is given by

$$L = \frac{1}{2}mv^2 + q\left(\phi - \vec{A}\,.\,\vec{v}\right),$$

where, $\phi\left(\vec{r}, t\right)$ and $\vec{A}\left(\vec{r}, t\right)$ are electric scalar potential and magnetic vector potential respectively.

Show that the Hamiltonian can be written as $H = \frac{1}{2m}\left(\vec{p} - q\vec{A}\right)^2 + q\phi$.

(b) If $[u, v]$ be the Poisson Bracket, then prove that $\frac{\partial}{\partial t}[u, v] = \left[\frac{\partial u}{\partial t}, v\right] + \left[u, \frac{\partial v}{\partial t}\right]$. 6+4

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